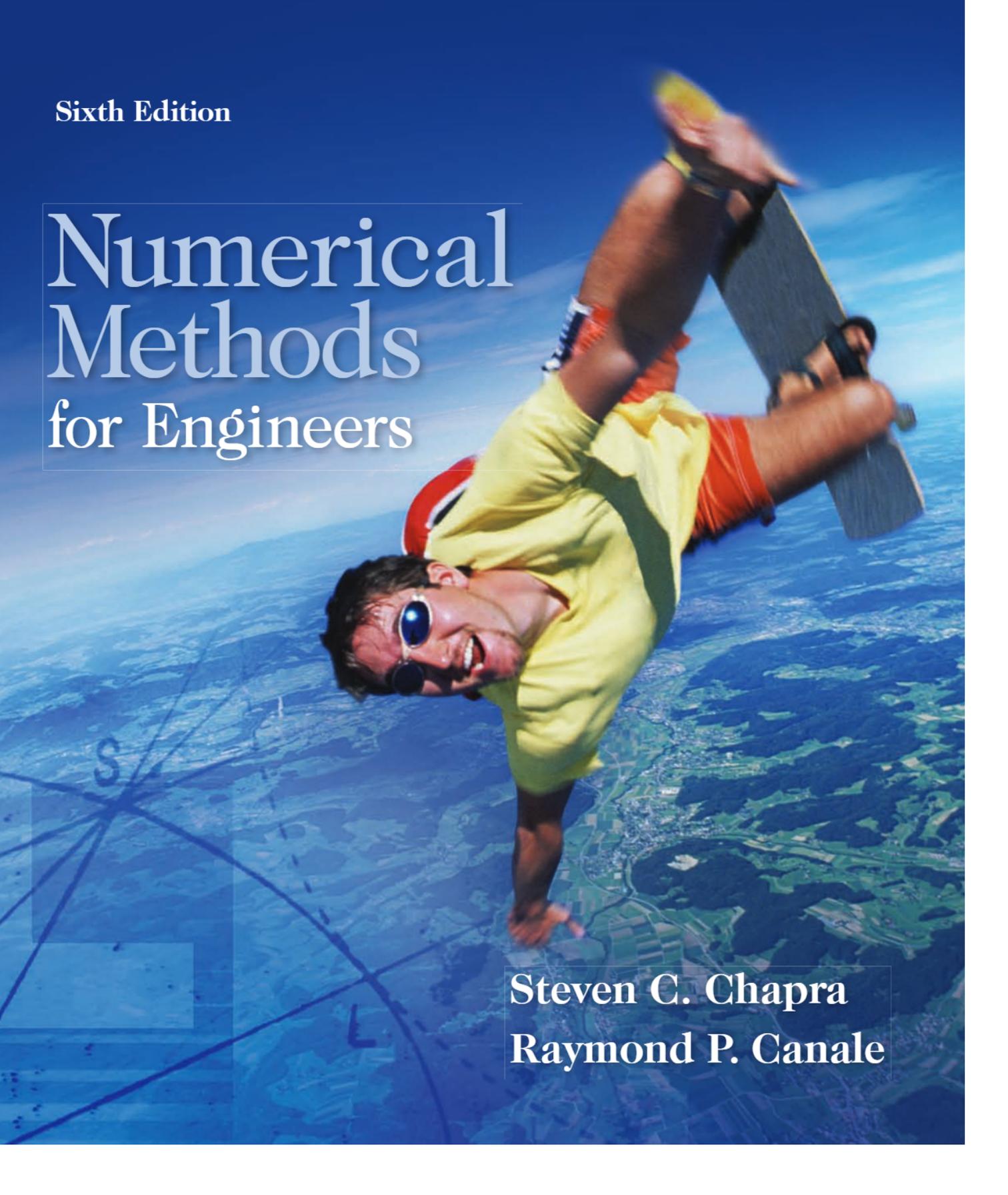


Sixth Edition

# Numerical Methods for Engineers

Steven C. Chapra  
Raymond P. Canale



# Numerical Methods for Engineers

SIXTH EDITION

**Steven C. Chapra**

Berger Chair in Computing and Engineering  
Tufts University

**Raymond P. Canale**

Professor Emeritus of Civil Engineering  
University of Michigan



**Higher Education**

Boston Burr Ridge, IL Dubuque, IA New York San Francisco St. Louis  
Bangkok Bogotá Caracas Kuala Lumpur Lisbon London Madrid Mexico City  
Milan Montreal New Delhi Santiago Seoul Singapore Sydney Taipei Toronto



## NUMERICAL METHODS FOR ENGINEERS, SIXTH EDITION

Published by McGraw-Hill, a business unit of The McGraw-Hill Companies, Inc., 1221 Avenue of the Americas, New York, NY 10020. Copyright © 2010 by The McGraw-Hill Companies, Inc. All rights reserved. Previous editions © 2006, 2002, and 1998. No part of this publication may be reproduced or distributed in any form or by any means, or stored in a database or retrieval system, without the prior written consent of The McGraw-Hill Companies, Inc., including, but not limited to, in any network or other electronic storage or transmission, or broadcast for distance learning.

Some ancillaries, including electronic and print components, may not be available to customers outside the United States.

This book is printed on acid-free paper.

1 2 3 4 5 6 7 8 9 0 VNH/VNH 0 9

ISBN 978-0-07-340106-5

MHID 0-07-340106-4

Global Publisher: *Raghothaman Srinivasan*

Sponsoring Editor: *Debra B. Hash*

Director of Development: *Kristine Tibbetts*

Developmental Editor: *Lorraine K. Buczek*

Senior Marketing Manager: *Curt Reynolds*

Project Manager: *Joyce Watters*

Lead Production Supervisor: *Sandy Ludovissy*

Associate Design Coordinator: *Brenda A. Rolwes*

Cover Designer: *Studio Montage, St. Louis, Missouri*

(USE) Cover Image: © *BrandX/JupiterImages*

Compositor: *Macmillan Publishing Solutions*

Typeface: *10/12 Times Roman*

Printer: *R. R. Donnelley Jefferson City, MO*

All credits appearing on page or at the end of the book are considered to be an extension of the copyright page.

MATLAB™ is a registered trademark of The MathWorks, Inc.

#### Library of Congress Cataloging-in-Publication Data

Chapra, Steven C.

Numerical methods for engineers / Steven C. Chapra, Raymond P. Canale. — 6th ed.  
p. cm.

Includes bibliographical references and index.

ISBN 978-0-07-340106-5 — ISBN 0-07-340106-4 (hard copy : alk. paper)

1. Engineering mathematics—Data processing. 2. Numerical calculations—Data processing 3. Microcomputers—Programming. I. Canale, Raymond P. II. Title.

TA345.C47 2010

518.02462—dc22

2008054296

www.mhhe.com



*To*

Margaret and Gabriel Chapra

Helen and Chester Canale

# CONTENTS

**PREFACE** xiv

**GUIDED TOUR** xvi

**ABOUT THE AUTHORS** xviii

## **PART ONE**

---

### **MODELING, COMPUTERS, AND ERROR ANALYSIS** 3

PT1.1 Motivation 3  
PT1.2 Mathematical Background 5  
PT1.3 Orientation 8

### **CHAPTER 1**

#### **Mathematical Modeling and Engineering Problem Solving** 11

1.1 A Simple Mathematical Model 11  
1.2 Conservation Laws and Engineering 18  
Problems 21

### **CHAPTER 2**

#### **Programming and Software** 25

2.1 Packages and Programming 25  
2.2 Structured Programming 26  
2.3 Modular Programming 35  
2.4 Excel 37  
2.5 MATLAB 41  
2.6 Mathcad 45  
2.7 Other Languages and Libraries 46  
Problems 47

### **CHAPTER 3**

#### **Approximations and Round-Off Errors** 52

3.1 Significant Figures 53  
3.2 Accuracy and Precision 55  
3.3 Error Definitions 56  
3.4 Round-Off Errors 62  
Problems 76

**CHAPTER 4**  
**Truncation Errors and the Taylor Series 78**

- 4.1 The Taylor Series 78
- 4.2 Error Propagation 94
- 4.3 Total Numerical Error 98
- 4.4 Blunders, Formulation Errors, and Data Uncertainty 103
- Problems 105

**EPILOGUE: PART ONE 107**  
PT1.4 Trade-Offs 107  
PT1.5 Important Relationships and Formulas 110  
PT1.6 Advanced Methods and Additional References 110**PART TWO****ROOTS OF EQUATIONS 113**

- PT2.1 Motivation 113
- PT2.2 Mathematical Background 115
- PT2.3 Orientation 116

**CHAPTER 5**  
**Bracketing Methods 120**  
5.1 Graphical Methods 120  
5.2 The Bisection Method 124  
5.3 The False-Position Method 132  
5.4 Incremental Searches and Determining Initial Guesses 138  
Problems 139**CHAPTER 6**  
**Open Methods 142**  
6.1 Simple Fixed-Point Iteration 143  
6.2 The Newton-Raphson Method 148  
6.3 The Secant Method 154  
6.4 Brent's Method 159  
6.5 Multiple Roots 164  
6.6 Systems of Nonlinear Equations 167  
Problems 171**CHAPTER 7**  
**Roots of Polynomials 174**  
7.1 Polynomials in Engineering and Science 174  
7.2 Computing with Polynomials 177  
7.3 Conventional Methods 180

- 7.4 Müller's Method 181
- 7.5 Bairstow's Method 185
- 7.6 Other Methods 190
- 7.7 Root Location with Software Packages 190
- Problems 200

## CHAPTER 8

### Case Studies: Roots of Equations 202

- 8.1 Ideal and Nonideal Gas Laws (Chemical/Bio Engineering) 202
- 8.2 Greenhouse Gases and Rainwater (Civil/Environmental Engineering) 205
- 8.3 Design of an Electric Circuit (Electrical Engineering) 207
- 8.4 Pipe Friction (Mechanical/Aerospace Engineering) 209
- Problems 213

## EPILOGUE: PART TWO 223

- PT2.4 Trade-Offs 223
- PT2.5 Important Relationships and Formulas 224
- PT2.6 Advanced Methods and Additional References 224

---

## PART THREE

### LINEAR ALGEBRAIC EQUATIONS 227

- PT3.1 Motivation 227
- PT3.2 Mathematical Background 229
- PT3.3 Orientation 237

## CHAPTER 9

### Gauss Elimination 241

- 9.1 Solving Small Numbers of Equations 241
- 9.2 Naive Gauss Elimination 248
- 9.3 Pitfalls of Elimination Methods 254
- 9.4 Techniques for Improving Solutions 260
- 9.5 Complex Systems 267
- 9.6 Nonlinear Systems of Equations 267
- 9.7 Gauss-Jordan 269
- 9.8 Summary 271
- Problems 271

## CHAPTER 10

### LU Decomposition and Matrix Inversion 274

- 10.1 LU Decomposition 274
- 10.2 The Matrix Inverse 283
- 10.3 Error Analysis and System Condition 287
- Problems 293

**CHAPTER 11**  
**Special Matrices and Gauss-Seidel 296**

- 11.1 Special Matrices 296
- 11.2 Gauss-Seidel 300
- 11.3 Linear Algebraic Equations with Software Packages 307
- Problems 312

**CHAPTER 12**  
**Case Studies: Linear Algebraic Equations 315**

- 12.1 Steady-State Analysis of a System of Reactors (Chemical/Bio Engineering) 315
- 12.2 Analysis of a Statically Determinate Truss (Civil/Environmental Engineering) 318
- 12.3 Currents and Voltages in Resistor Circuits (Electrical Engineering) 322
- 12.4 Spring-Mass Systems (Mechanical/Aerospace Engineering) 324
- Problems 327

**EPILOGUE: PART THREE 337**

- PT3.4 Trade-Offs 337
- PT3.5 Important Relationships and Formulas 338
- PT3.6 Advanced Methods and Additional References 338

**PART FOUR****OPTIMIZATION 341**

- PT4.1 Motivation 341
- PT4.2 Mathematical Background 346
- PT4.3 Orientation 347

**CHAPTER 13**  
**One-Dimensional Unconstrained Optimization 351**

- 13.1 Golden-Section Search 352
- 13.2 Parabolic Interpolation 359
- 13.3 Newton's Method 361
- 13.4 Brent's Method 364
- Problems 364

**CHAPTER 14**  
**Multidimensional Unconstrained Optimization 367**

- 14.1 Direct Methods 368
- 14.2 Gradient Methods 372
- Problems 385

**CHAPTER 15**  
**Constrained Optimization 387**

- 15.1 Linear Programming 387
- 15.2 Nonlinear Constrained Optimization 398
- 15.3 Optimization with Software Packages 399
- Problems 410

**CHAPTER 16**  
**Case Studies: Optimization 413**

- 16.1 Least-Cost Design of a Tank (Chemical/Bio Engineering) 413
- 16.2 Least-Cost Treatment of Wastewater (Civil/Environmental Engineering) 418
- 16.3 Maximum Power Transfer for a Circuit (Electrical Engineering) 422
- 16.4 Equilibrium and Minimum Potential Energy (Mechanical/Aerospace Engineering) 426
- Problems 428

**EPILOGUE: PART FOUR 436**

- PT4.4 Trade-Offs 436
- PT4.5 Additional References 437

**PART FIVE**

---

**CURVE FITTING 439**

- PT5.1 Motivation 439
- PT5.2 Mathematical Background 441
- PT5.3 Orientation 450

**CHAPTER 17**  
**Least-Squares Regression 454**

- 17.1 Linear Regression 454
- 17.2 Polynomial Regression 470
- 17.3 Multiple Linear Regression 474
- 17.4 General Linear Least Squares 477
- 17.5 Nonlinear Regression 481
- Problems 484

**CHAPTER 18**  
**Interpolation 488**

- 18.1 Newton's Divided-Difference Interpolating Polynomials 489
- 18.2 Lagrange Interpolating Polynomials 500
- 18.3 Coefficients of an Interpolating Polynomial 505
- 18.4 Inverse Interpolation 505
- 18.5 Additional Comments 506
- 18.6 Spline Interpolation 509
- 18.7 Multidimensional Interpolation 519
- Problems 522

**CHAPTER 19****Fourier Approximation 524**

- 19.1 Curve Fitting with Sinusoidal Functions 525
- 19.2 Continuous Fourier Series 531
- 19.3 Frequency and Time Domains 534
- 19.4 Fourier Integral and Transform 538
- 19.5 Discrete Fourier Transform (DFT) 540
- 19.6 Fast Fourier Transform (FFT) 542
- 19.7 The Power Spectrum 549
- 19.8 Curve Fitting with Software Packages 550
- Problems 559

**CHAPTER 20****Case Studies: Curve Fitting 561**

- 20.1 Linear Regression and Population Models (Chemical/Bio Engineering) 561
- 20.2 Use of Splines to Estimate Heat Transfer (Civil/Environmental Engineering) 565
- 20.3 Fourier Analysis (Electrical Engineering) 567
- 20.4 Analysis of Experimental Data (Mechanical/Aerospace Engineering) 568
- Problems 570

**EPILOGUE: PART FIVE 580**

- PT5.4 Trade-Offs 580
- PT5.5 Important Relationships and Formulas 581
- PT5.6 Advanced Methods and Additional References 583

**PART SIX****NUMERICAL  
DIFFERENTIATION  
AND  
INTEGRATION 585**

- PT6.1 Motivation 585
- PT6.2 Mathematical Background 595
- PT6.3 Orientation 597

**CHAPTER 21****Newton-Cotes Integration Formulas 601**

- 21.1 The Trapezoidal Rule 603
- 21.2 Simpson's Rules 613
- 21.3 Integration with Unequal Segments 622
- 21.4 Open Integration Formulas 625
- 21.5 Multiple Integrals 625
- Problems 627